Electron - Proton Scattering as a Probe of Nucleon Structure

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The problem of electron-proton scattering is handed over both the elastic and inelastic scattering. Two models are presented in this sense. The first, depends on the multi photon exchange ladder diagram, where the transition matrix is expanded in multi steps form. The second model uses the multi peripheral mechanism developed for the electromagnetic field. It allows the particle production in the inelastic scattering processes. An iterative procedure is found and inserted in a Monte Carlo program to reproduce the differential cross section of the reaction. The comparison with the experimental data shows bid fair in most cases.

I. INTRODUCTION

Recent experiments for electron-proton (ep) scattering, SLAC-E-136 [1], DESY-HERA-HE [2], SLAC-E-133 [3] and others [4], played important role in probing the nucleon structure and to reveal the dynamic mechanism of the electron interactions inside the nucleon bag. Many trials have been done before by a wide variety of empirical and theoretical models, ranging from form factor scaling [5], and vector meson dominance [6] to quark- parton models [7] and perturbative QCD [8]. An eikonal optical picture [9-10] was used, based on multiple scattering of the incident electron with the constituent valance quarks of the target hadron, assuming different forms of the (electron-quark) binary wave functions.

In this article we shall deal with the problem of elastic and inelastic collisions of the electron with proton from a different point of view. In the following, we present two pictures for the (ep) scattering. The first is a scattering in view of multi-photon exchange mechanism (MPEM) which is very convenient for the elastic scattering in a wide range of momentum transfer square Q^2. On the other hand, the multi-peripheral model (MPM) is relevant to the inelastic and deep inelastic scattering and so works properly for the problems of multi particle production. The article structure goes as follows. In section 2, we present the postulates of the MPEM, the formulation and the scenario of the MPM are given in section 3. The Monte Carlo generators are summarized in section 4. Finally results and discussion are given in section 5, followed by conclusive remarks.

II. THE MULTI PHOTON EXCHANGE MODEL (MPEM)

It is assumed that elastic scattering of electrons on proton proceeds via multi step processes represented by ladder diagrams [11]. We proceed on the bases of the Feynman formalisms, assuming electromagnetic interaction acting at each vertex of the ladder diagram Fig.(1). Expanding the transition matrix T of the (ep) scattering in terms of transition ladder diagrams $T^{(n)}$ so that,

$$T = \prod_{i}^{n-1} T_i \tag{1}$$

$\{c^n\}$ and $\{T^{(n)}\}$ are the coefficients of expansion and transition matrices of the adder diagram of order n. $T^{(n)}$ has the form,

$$T^{(n)} = \int \cdots \int \prod_{1}^{n-1} dk_j \frac{V_{j+1,j} V_{j,j-1}}{k - k_j + i\epsilon}. \tag{2}$$

Where the factor $\frac{1}{k-k_j+i\epsilon}$ stands for the Green's propagator of the virtual intermediate state number j. $V_{j,j-1}$ is the transition probability from the state $j-1$ to j, so that, $V_{j,j-1} =< \phi_j|V|\phi_{j-1} >$. The screening Coulomb field working at each vertex has the form, $V = -U_0 \frac{e^{-\alpha r}}{r}$ so that, each internal integration, $V_{kk'}$ has the form;

$$< k|V|k' >= -U_0[2\pi^2(\alpha^2 + |k - k'|^2)]. \tag{3}$$

Hence, a one step ladder diagram has a transition matrix $T^{(1)}$,

$$T^{(1)} = -U_0[2\pi^2(\alpha^2 + |k_i - k'_f|^2)] \tag{4}$$

k_i and k_f are the momentum of the initial and final states. Similarly, the two step and the three step ladders will have the forms,

$$T^{(2)} = -2\pi^2 U_0^2 \int \frac{d\,\bar{k}_1}{(k_1 - k^2 - i\epsilon)(\alpha^2 + |k_i - k'_1|^2)(\alpha^2 + |k_1 - k'_f|^2)} \tag{5}$$

$$T^{(3)} = \int\int \frac{(U_0^3/4\pi^4)d\,\bar{k}_1\;d\,\bar{k}_2}{(k - k_2^2 - i\epsilon)(\alpha^2 + |k_2 - k'_1|^2)(\alpha^2 + |k_2 - k'_f|^2)(k - k_1^2 - i\epsilon)}.$$

$$\frac{1}{(k - k_1^2 - i\epsilon)(\alpha^2 + |k_i - k'_1|^2)} \tag{6}$$

The integrals in Eqs.(5,6) are carried out by the Dalitz integrals [12]. The expansion coefficients $\{c_i\}$ in Eq.(1) are determined by fitting with the experimental data to estimate the statistical weight factor of each ladder diagram contributing the reaction. According to the usual formalism for the dynamics of the particle reactions [13], the phase space integral is an integration of the square of the transition matrix
$T(\{k_i\})$ over a set of allowed values $\{k_i\}$.

$$I_n(s) = \int \prod_i^n \frac{d^3k_i}{2E_i}\delta^4(k_a + k_b - \sum_{i=1}^n k_i)|T\{k_i\}|^2 \tag{7}$$

k_a, k_b are the momenta of the interacting particles in the initial state, while $\{k_i\}$being the momenta of the particles in the final state. For the problem under consideration of the elastic scattering, where only two particles in the final state $n = 2$, with the normalization convention, the reaction cross section for $k_a + k_b \rightarrow k_1 + k_2$ is

$$\sigma(s) = \frac{1}{8\pi^2\lambda^{1/2}(s, m_a^2, m_b^2)} \int \frac{d^3k_1}{2E_1}\frac{d^3k_2}{2E_2}\delta^4(k_a + k_b - k_1 + k_2)|T\{k_1, k_2\}|^2 \tag{8}$$

λ is a standard function defined as,$\lambda(x, y, z) = (x - y - z)^2 - 4yz$. One may immediately write the differential cross section in the center of mass system CMS

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{k_1}{k_a}|T|^2 \tag{9}$$

It is more convenient to use the invariant cross section,

$$\frac{d\sigma}{dt} = \frac{1}{16\pi\lambda(s, m_a^2, m_b^2)}|T|^2, \qquad t = -Q^2 \tag{10}$$

Eqs. (9),(10) as well as Eq.(1) are in a relevant form to compare with the experimental data.

III. THE MULTI PERIPHERAL MODEL (MPM)

In this section, we shall deal with the problem of the particle production in the (ep) inelastic scattering in view of the multi peripheral collision [14]. A factorizable transition matrix T is assumed in the form,

$$T = \prod_i^{n-1} T_i \tag{11}$$

Each particle in the final state is produced at a specific peripheral surface as shown by the Feynman diagram Fig.(2) with electromagnetic transition matrix T_i, may be written in a suitable parametric form,

$$T = \frac{1}{\alpha_i + t_i} \tag{12}$$

where t_i is the four vector momentum transfer square at the i^{th} peripheral surface. α_i is the electromagnetic peripheral parameter characterizing the surface number i, and determine to conserve the total energy. The advantage of this technique is to reduce the many body problem into $(n-1)$ iterative diagrams, each of them has only two particles in the final state. For example, the i^{th} diagram has two particles in the final state, the first one is the particle number $(i+1)$, and the other one has an effective mass M_i, equivalent to the rest of the i-particles of the system. The square of the 4-vector momentum transfer t_i is kinematically calculated as,

$$t_i = (p_a^{(i)} - p_1 - .... - p_i)$$

$$t_i = m_{i+1}^2 + M_i^2 - 2E_a^{(i)} k_i^0 - 2P_a^{(i)} K_i \cos\theta_i \tag{13}$$

m_{i+1} is the rest mass of the particle number $(i+1)$ produced in the i^{th} iteration. K_i and k_i^0 are the 3-vector momentum and the total energy of the effective mass M_i. and are the corresponding figures for the leading particle acting at the i^{th} peripheral surface. The recursion relation of $P_a^{(i)}$is given by,

$$P_a^{(i)} = \lambda^{1/2}(M_i^2, t_i, m_a^2)/2M_i \tag{14}$$

The leading particle for the first peripheral surface is given by

$$P_a = P_a^{(n)} = \lambda^{1/2}(s, m_a^2, m_b^2)/2\sqrt{s} \tag{15}$$

The multi peripheral parameters $\{\alpha_i\}$ play important role in converging the particles in phase space and consequently, control the energy of the particles in final state. So that the values of $\{\alpha_i\}$ are adjusted to conserve the total energy. The energy E_i of the particle number i is related to its rapidity y_i through the relation,

$$E_i = m_t \cosh(y_i)$$

$$m_t = \sqrt{P_t^2 + m_i^2} \tag{16}$$

so that the total energy of the particles in the final state is

$$\zeta_i^n = \frac{1}{\sigma_n} \int m_t \cosh(y)(d\sigma/dy)dy \tag{17}$$

ζ_i^n which is a function of the parameters $\{\alpha_i\}$ should be compared with the total center of mass energy$\sqrt{s}$ of the initial state. We first start with $n = 2$ to get α_1, which is inserted again in the case $n = 3$ to get α_2 and so on. These are repeated sequentially to get the values of the rest parameters up to α_{n-1}. The values $\{\alpha_i\}$ depend on the particle multiplicity n rather than the energy $\sqrt{s}$. The phase space integral $I_n(s)$ is then calculated as in Eq.(7) after transforming the integral variables from k_i to t_i,

$$I_n(s) = \frac{(2\pi)^{n-1}}{2M_n} \prod_{i=3}^{n} \frac{1}{4P_a^{(i)}} \int_{\mu_i}^{M_i - m_i} dM_{i-1} \int_{t_{i-1}^-}^{t_{i-1}^+} \frac{1}{t_{i-1} + \alpha_{i-1})^2} dt_{i-1}.$$

$$\frac{1}{4P_a^{(2)}} \int_{t_1^-}^{t_1^+} \frac{1}{t_1 + \alpha_1)^2} dt_1 \tag{18}$$

$$\mu_i = \sum_{j=1}^{i} m_j \tag{19}$$

$t_i^{\pm}$ are the upper and lower limits of t_i corresponding to $cos(\theta_i) = \pm 1$.

$$t_i^{\pm} = m_a^2 + m_b^2 - 2E_a E_b \pm 2P_a P_b \tag{20}$$

Multiple integrals in Eq.(18) are carried out using a Monte Carlo program, through which all possible distributions of the physical quantities are easily found.

IV. MONTE CARLO

A Monte Carlo program GENE2 [15] is developed to simulate events according to the multi peripheral diagrams. It includes 3-generators. The generator $Gn(s)$ for the multiplicity of particles in the final state, the generator $GM(s,n)$ for the invariant masses produced at the $n-1$ possible peripheral surfaces and finally the dynamic generator $Gt(s,n,M)$ which generates the values of the momentum transfer square t_i of the i^{th} surface according to electromagnetic transition matrix. The program GENE2 executes the 3- generators in ordered sequences, and simulates all the kinematical variables of the n- particles in the final state.

A general form of the algorithm used to generate an event x with probability $f(x)$ is,

$$r = \int_a^x f(x)dx / \int_a^b f(x)dx \tag{21}$$

Where a and b are the boundaries of the physical region at which x is defined. r is the default random number with uniform distribution between the limits $\{0,1\}$.

A. The multiplicity generator Gn(s)

It is assumed that the multiplicity distribution $P(n)$ has a Gaussian form [16],

$$P(n) = \frac{1}{\sqrt{2\pi}\sigma} \exp[-(n-\bar{n})^2/\sigma^2] \tag{22}$$

where $\bar{n}$ is the average multiplicity which depends on the center of mass energy $\sqrt{\sigma}$ and σ is the dispersion.

$$\bar{n} = a\log(s) + b \qquad , \sigma = c\ \bar{n} + d \tag{23}$$

the integration limits extend from zero to infinity. This makes Eq.(21) read;

$$r = Erf\,(n - \bar{n}) \tag{24}$$

$Erf(x)$ is the error function. The solution n of Eq. (24) defines the multiplicity generator of the reaction.

B. The invariant mass generator GM(s,n)

According to the multi peripheral diagrams Fig.(2), it is assumed that the peripheral surface number i, produces the particle number $i+1$ and the invariant mass M_i, equivalent to the effective mass of the system of the rest of the $i-$particles. The value of M_i satisfies the relation,

$$\sum m_i \leq M_i \leq M_{i+1} \tag{25}$$

so that the algorithm of the generator GM(s,n) is,

$$M_i = \sum_j^i m_j + r(M_{i+1} - \sum_{j=1}^i m_j) \tag{26}$$

For a system of n-particle final state, we start with $M_n = \sqrt{s}$, then generate the value of M_{n-1}. Eq.(26) is the recursion relation to generate all values of M_i, $\quad i = n-1, ..., 2$.

C. The dynamic generator Gt(s,n,M)

The algorithm used to generate the values of the square of the momentum transfer t_i should simulate a probability distribution which is proportional to the square of the transition matrix T_i defined by Eq.(12). Inserting this in Eq.(21), we get the t_i generator as,

$$t_i = \{r[(t_i^+ + \alpha_i)^{-1} - (t_i^- + \alpha_i)^{-1} + (t_i^+ + \alpha_i)^{-1}\}^{-1} - \alpha_i \tag{27}$$

V. RESULTS AND DISCUSSION

A. The elastic scattering

The problem of elastic scattering is treated in this article using the ladder diagrams of multi steps. The transition matrix $T^{(n)}$ is calculated for ladder diagrams for $n = 1, 2$ and 3 using the Dalitz and Feynman integrals. The results are demonstrated in Fig.(3). The potential field acting at each vertex of the diagram is assumed to be of electromagnetic nature with screening factor due to the pion current inside the proton target. The screening effect limits the infinite range of the coulomb potential. It is found that the terms of the transition matrix $T^{(n)}$ form a converging series. The first term of which decreases rapidly with the momentum transfer square $t = -Q^2$. while the higher order terms are slowly varying functions. The asymptotic behavior of $T^{(n)}$ follows a power law at extremely high energy.

The expansion coefficients $\{c_i\}$ are determined by the fitting method with the SLAC experimental data [4] in the range $t \backsim 1 - 3(GeV/c)^2$. The results are shown in Fig.(4).

It is found that only two terms of the series are sufficient to represent the reaction. The maximum probable diagram corresponds to $n = 1$. On the other hand, the SLAC-E-136 experiment, Fig.(5) in the momentum transfer range $t \backsim 3 - 31(GeV/c)^2$ needs more terms of the series, with a most probable diagram corresponds also to $n = 1$. The values of the expansion coefficients are given in Table (1).

Table(1) The branching ratios of multi photon exchanged in ep collisions.

Reaction	1-Step	2_Steps	3-Steps
Ref.[3]	91	9	-
SLAC-E-136	86	12	2

Figure (4) and (5) show good agreement between the experimental data and the prediction of the multi photon exchange mechanism. It is clear that the reactions of high momentum transfer needs more terms of ladder diagrams.

B. The inelastic scattering.

Here we use the iterative multi peripheral diagram as in Fig.(2-a,b). The advantage of this is to simulate a complete inclusive reaction. We used the Monte Carlo program GENE2 to generate the n-particles in the final state. The multi peripheral parameters α_i, $i = 1, 2, \cdots, n-1$ are determined to conserve the total center of mass energy $\sqrt{s}$. The values of the parameters $\{\alpha_i\}$ depend not only on $\sqrt{s}$ but also on the degree of peripherality i. The first few peripheral surfaces possess parameter values which confine the particle production in very narrow cone. The production cone angle gets wider for higher order surfaces. The values of α_i are displayed in Fig.(6) as a function of the multiplicity n, of the number of particle in the final state for electron lab energies $50, 100$ and $200 GeV$. In all cases the value of α_i decreases slowly with n, up to a critical multiplicity nc after which a sudden drop is observed. A parametric relation is obtained for α_i as a function of the electron lab energy E and n as,

$$\alpha_{n-1} = (0.226 - 0.015E + 0.0003E^2)n + 0.441 \log(E) - 2.29, \quad n < n_c.$$

$$\alpha_{n-1} = (0.0019E - 0.8685) \exp[(-.000688E + 0.327276)n], \quad n > n_c$$

$$n_c = 0.4846E + 1.0769 \tag{28}$$

The critical value n_c corresponds to the peripheral surface at which enough energy is transferred, that is sufficient to make phase transition from the nuclear matter to the quark gluon state. The model is applied to the data of the experiment SLAC-E-133 corresponding to electron lab energies $9.744, 12.505, 15.730, 18.476$ and $20.999 GeV$. Fig.(7) shows the rapidity distributions for particles produced at $9.744 GeV$ electron lab energy as calculated by (MPM) at peripheral surfaces $n = 2, 3, 5$ and 10. The result for $n = 2$ (two particle final state) shows two non symmetric clear peaks corresponding to forward and backward emission. The electron peak is relatively narrower than the proton one . This case represents the most peripheral collision with low momentum transfer. As the multiplicity increases the two peaks get broader and interfere through each other. They completely interfere and form only one peak at the case of high multiplicity $n = 10$ corresponding to high momentum transfer. This represents the most central collision. i.e. collision between the electron and the proton core. The missing mass of the recoil nucleon is also calculated through the Monte Carlo program GENE2, according to the relation,

$$W^2 = M_p^2 + 2M_p(E - E') - 4EE' \sin^2 \frac{\theta}{2} \tag{29}$$

The differential cross section $d\sigma/d\Omega$ as a function of W^2 is compared in Fig.(8) with the experimental data of energies $9.744, 12.505$ and $15.730 GeV$. Fair agreement is obtained for the first reaction only. The deviation increases as the electron energy increases. This may be due to the fact that the model in hand has ignored the relative motion of the core inside the nucleon target. This relative motion let the recoil nucleon not has a unique value of missing mass, but instead, it posses a Gaussian -like distribution around $W^2 = M_p^2$. The peak is wider at higher t.

VI. CONCLUSION

1- Data of elastic scattering may be reproduced in a wide range of the energy transferred using the (MPEM) with an expandable transition matrix representing ladder diagrams.

2- A transition matrix of order n, may be approximated as the power form of the one step ladder diagram at extremely high energy.

3- The power series of the transition matrix contains number of terms increasing with the energy transferred of the reaction.

4- Inelastic ep scattering are successfully described by the multi peripheral model.

5- The peripheral parameters are the dynamic parameters, which control the convergence of the particles in phase space. A critical value of peripheral parameter determines the critical surface at which phase transition may occur from the nuclear matter density to the quark gluon phase.

6- The rapidity distribution of the particles in the final state shows two peaks representing the forward and backward production. The peaks interfere together as the multiplicity increases in the final state.

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[1] R.G. Arnold, et al., Phys. Rev. Lett. 57 (1986) 174.
[2] H1 Collaboration, Phys. Lett. B. 328 (1994) 176.
[3] S. E. Rock, et al., Phys. Rev. D 46 (1992) 24.
[4] P. E. Bosted, et al., Phys. Rev. Lett. 68 (1992) 3841. J. Gomez, et al., Phys. Rev. D 49 (1994) 4348.
[5] T. Narita, Prog. Theor. Phys. 42 (1969) 1336.
[6] G. Hohler, et al., Nucl. Phys. B 114 (1976) 505. J. G. Korner and M. Kuroda, Phys. Rev. D16 (1977) 2165.
[7] S.D. Drell and T.M.Yan, Phys. Rev. Lett. 24 (1970) 181. S.J. Brodsky and B.Chertock, Phys. Rev. D14 (1976) 3003.
[8] C.R. Ji and A.F. Sill Phys. Rev.D 34 (1986) 3350.
[9] M.T. Hussein, 24th International Cosmic Ray Conference, Roma, HE, (1995) 135.
[10] M. K. Hegab , M. T. Hussein and N.M. Hassan, Z. Phys. 336 (1990) 345.
[11] G. Aznauryan and N.L. Ter-Isakyan, Sov. J. Nucl. Phys. 31 (1980) 871.
G.F. Chew and S. Manelstam, Phys. Rev. 11(1960) 467.
L.L. Frankfurt and M.I. Strikman, Phys. Reports. 76C (1981) 215.
[12] R.H.Dalitz,Proc.Roy.Soc.A206(1951)509,R.P.Feynman,Phys.Rev.76 (1949)769.
[13] M.T. Hussein ,N.M. Hassan and M.K. Hegab ,Can. J. Phys. 63 (1985) 1449.
[14] M. T. Hussein , IL Nuovo Cimento 106A , 481 (1993).
[15] M.T. Hussein, "Monte Carlo Simulation for Hadron Fragmentation at High Energies". Invited Talk, First International Conference. On Basic Science and Advanced Technology. Nov. 9 -12, (1996). Assiut, Egypt.
[16] M.T. Hussein and A. El-Naghy, 18th International Cosmic Ray Conference, Bangalore India (1983) 1.